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EFFECT OF TURBULENCE ON THE PARTICLE SETTLING VELOCITY IN THE NONLINEAR DRAG RANGE

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Abstract—Particle settling velocity in the nonlinear drag range is investigated using a Monte Carlo simulation for particles in a low Reynolds number, isotropic, Gaussian, pseudo turbulence. The settling velocity is affected by both the trajectory bias, which enhances the settling velocity, and the nonlinearity of the drag associated with the turbulence, which reduces the settling velocity. The effect of the trajectory bias is important in an almost frozen turbulence when the settling velocity is comparable to the turbulence and particle motion in the creeping-flow regime. For a nonfrozen turbulence, the effect of the trajectory bias on the settling velocity may be overwhelmed by the effect of the nonlinear drag associated with the turbulence. The ensemble average of the second invariant of the turbulence deformation tensor, $\langle \mathrm{II_d} \rangle$, along the particle trajectories is obtained to characterize statistically the trajectory bias and the correlation between the particle concentration and turbulence structure. $\langle \mathrm{II_d} \rangle$ attains its maximum value at zero settling for a given particle inertia. The effect of increasing the settling velocity leads to an exponentially decreasing $\langle \mathrm{II_d} \rangle$ for a large settling rate, and hence a significant reduction in the trajectory bias and the concentration–structure correlation. For very small or large particle inertia, $\langle \mathrm{II_d} \rangle$ vanishes.

Key Words: settling velocity, turbulence, nonlinear drag, trajectory bias

1. INTRODUCTION

It is known that a particle possesses a different gravitational settling velocity in an oscillating fluid than in a still fluid, in general. Tunstall & Houghton (1968) and Schöneborn (1975) showed experimentally a reduction in the settling velocity of a solid particle in an oscillating liquid. Hwang (1985) and Ikeda & Yamasaka (1989) showed, through detailed analyses, that for particle motion in the nonlinear drag range the settling velocity, V_T, in the direction of gravity, $\mathbf{e}_1$, of the particle is *lower* than in a still liquid if the liquid is oscillating in the direction parallel to settling. Murray (1970) found that V_T is smaller in a turbulent liquid (generated by two oscillating grids) than in a still liquid. The reduction in V_T was shown to result from the nonlinearity in the drag law (Hwang 1985). The effect of the turbulence on the settling rate of heavy particles in the nonlinear drag range was investigated analytically by Mei (1990) using a low Reynolds number turbulence energy spectrum (Kraichnan 1970). It was shown that the nonlinearity in the drag law can cause a significant reduction in V_T for particles in turbulence.

Nielsen (1984) analyzed the trajectory of sediment particles in a vortex flow $(u, v) = \omega(-y, x)$, in which ω is the angular velocity and (u, v) are the velocity components in the Cartesian coordinates (x, y). It was shown that the sediment particles can be suspended permanently in such a vortex flow. Nielsen thus attributed the reduction in the settling velocity observed in Murray's experiment to the "vortex trapping", i.e. the ordered turbulent eddies keep particles within the eddies and reduce the settling velocity of the particles in liquid, rather than to the nonlinearity in the drag law.

Maxey & Corssin (1986) studied the effect of the spatial structure of the fluid flow on the settling velocity of heavy particles by examining the motion of Stokesian particles in randomly oriented, periodic cellular flow fields. Contrary to the "vortex trapping" of the settling particles in vortex flow, an increase in the settling velocity was observed for most cases in cellular flow. Using the Stokesian drag law, Maxey (1987) found that the particle settling velocity is enhanced by turbulence through a Monte Carlo simulation and through asymptotic analyses of particle motion in a pseudo Gaussian turbulence. It was shown that the increase in V_T was caused by the bias of the particle trajectories towards regions of high strain rate and low vorticity in a statistically homogeneous turbulent flow. Squires & Eaton (1990) tracked a large number of particles in a randomly forced isotropic turbulence generated using a direct numerical simulation (DNS). They have shown that particles have a tendency to accumulate in regions of high strain rate or low vorticity, which supports Maxey's (1987) finding for the Stokesian particle in a more realistic turbulence. Yeh & Lei (1991) reported an increase in the settling velocity for particles in a decaying homogeneous isotropic turbulence generated by a large eddy simulation. The change is slightly less than that of Maxey (1987) for a given combination of particle inertia and settling velocity (in still fluid). More recently, Wang & Maxey (1993) investigated the change in the settling velocity of aerosol particles in a forced isotropic turbulence generated by DNS. An even larger relative increase in the settling velocity was found than in the earlier investigation (Maxey 1987). Thus, it is clear that, while an ordered vortex flow does trap sediment particles as shown by Nielsen (1984), the "vortex trapping" cannot be the mechanism that is responsible for the reduction in the particle settling velocity in random turbulent flows. The bias in the particle trajectory actually enhances, rather than reduces, the particle settling velocity in a random velocity field. Nevertheless, it must be pointed out that the "vortex trapping" could be in force, such as bubbles in turbulent liquid flow, due to the added-mass force and the fluid acceleration. The bubble rising velocity should be reduced (Nielsen 1993). In this paper, only heavy particles are considered, so that particles will tend to accumulate in regions of high strain rate or lower vorticity. We shall focus on the settling of particles in a random turbulence rather than ordered flows, such as vortex or cellular flow, which are far from realistic turbulence.

To recapitulate, there are two opposing effects of the turbulence structure on the particle settling velocity: (i) the trajectory bias causes particles to collect in regions of high strain rate with a higher probability of greater downward fluid velocity; and (ii) the nonlinearity in the drag law and the turbulent fluctuation increase the effective particle time constants, which leads to an effective reduction in the settling velocity. It is clear that as the nonlinearity decreases, the second mechanism becomes unimportant while the first mechanism could be important. Thus, the focus of this paper shall be on the nonlinear drag range in which the second mechanism may be more important than the first one. In the works of Hwang (1985), Ikeda & Yamasaka (1989) and Mei (1990), the effect of the trajectory bias, the existence of which is clearly shown in Maxey (1987), was not included. However, numerous experimental data (Hwang 1985) have indicated that the settling velocity in a time-varying flow field is reduced compared with that in a still fluid. Maxey (1987) only considered a Stokesian particle, which rules out the mechanism for reducing the settling velocity. In Squires & Eaton (1990), correlations between the contours of the instantaneous concentration and that of the instantaneous second invariant of the turbulence deformation tensor, II_d, were demonstrated. However, the effect of the settling velocity on the trajectory bias or the "preferential concentration" was not examined and the effect of the particle inertia was not examined thoroughly due to the computational cost. In Yeh & Lei (1991), a nonlinear drag law was used in general; but the contributions from the nonlinear drag and the trajectory bias to the settling velocity were not examined separately. The result reported was simply a confirmation of the trajectory bias effect on the settling velocity in a different turbulence. Wang & Maxey (1993) included a discussion on the effect of the drag nonlinearity on the settling rate in their DNS based on the data obtained for $Re_p \sim 1$, in which Re_p is the particle Reynolds number (see [4] below). The opposing effects of the drag nonlinearity and the trajectory bias were demonstrated; but the effect of the drag nonlinearity is not explored thoroughly.

The purpose of this paper is to investigate, and to clarify, the effects of these opposing mechanisms by using a Monte Carlo simulation for particle motion in a Gaussian, pseudo turbulence with both linear and nonlinear drag laws. In section 2, the simplified equation for the motion of heavy particles is given and the effect of the nonlinear drag on the particle settling velocity in turbulence is illustrated analytically in the large settling limit. The Monte Carlo simulation procedure for particle motion is described briefly. In section 3.1, an analytical procedure for evaluating the particle dispersion and settling velocity is outlined following Mei (1990). The settling rate λ evaluated under five different conditions is defined. The effects of these two opposing mechanisms on the settling of heavy particles are examined in detail in section 3.2; the contributions from the drag nonlinearity and the trajectory bias to the settling velocity are assessed. In section

4, the effect of the particle inertia and settling velocity on the trajectory bias is examined quantitatively by computing the ensemble average of the second invariant of the turbulence deformation tensor, $\langle \mathrm{II}_d \rangle$, on the particle trajectories. It is shown that increasing settling velocity results in an exponentially decreasing $\langle \mathrm{II}_d \rangle$, a decrease in the trajectory bias and, therefore, a rapid destruction of the correlation between the concentration and turbulence structure. At zero settling, the largest $\langle \mathrm{II}_d \rangle$ occurs at some finite particle inertia.

2. PARTICLE MOTION IN ISOTROPIC GAUSSIAN TURBULENCE

The following form of the particle dynamic equation is used to study the effect of the turbulence structure on the settling velocity:

$$\tfrac{4}{3}\pi a^3 \rho_p \frac{d\mathbf{V}}{dt} = \tfrac{4}{3}\pi a^3 \rho_p g \mathbf{e}_1 + \phi 6\pi\rho_f \nu a(\mathbf{u} - \mathbf{V}) \qquad [1]$$

and

$$\frac{d\mathbf{Y}}{dt} = \mathbf{V} = V_T \mathbf{e}_1 + \mathbf{v}. \qquad [2]$$

The drag law is taken from Clift *et al.* (1978) as

$$\phi = C_D \frac{\mathrm{Re}_p}{24} = 1 + b\,\mathrm{Re}_p^n; \quad b = 0.15, \quad n = 0.687. \qquad [3]$$

In [1]–[3], $\mathbf{u}(\mathbf{x}, t)$ and $\mathbf{V}(t)$ are the velocities of the fluid and the particle relative to the mean uniform fluid flow, $\mathbf{v}$ is the particle turbulent velocity, V_T is the actual settling velocity of the particle in the gravitational direction $\mathbf{e}_1$, $\mathbf{Y}(t)$ represents the trajectory of the particle in a frame moving with the mean flow, g is the gravitational acceleration, ρ_p and ρ_f are the particle and fluid densities, ν is the kinematic viscosity of the fluid and C_D is the standard drag coefficient at steady state. The instantaneous particle Reynolds number is defined, based on the diameter of the particle, as

$$\mathrm{Re}_p = \frac{|\mathbf{u} - \mathbf{V}|2a}{\nu}. \qquad [4]$$

The effect of the history force on the particle dispersion in the Stokesian drag range was found to be small (Mei *et al.* 1991); the force resulting from the undisturbed fluid stress (in the absence of particles) and the added-mass force are of high order in comparison with the history force for a particle with a small Stokes number $\epsilon = (\omega_0 a^2/2\nu)^{1/2}$, in which ω_0 is a typical frequency of the turbulence. These unsteady forces are, therefore, not included in the present investigation.

The reduction in the settling velocity due to the nonlinearity of the drag law in an oscillating fluid or in a turbulence can be exemplified by taking the ensemble average, denoted by $\langle\ \rangle$, after the initial transient, on both sides of [1]:

$$\tfrac{4}{3}\pi a^3 \rho_p g + 6\pi\rho_f \nu a < (\mathbf{u} - \mathbf{V})\phi > = 0, \qquad [5a]$$

which gives

$$V_T + b\langle \mathrm{Re}^n(\mathbf{V} - \mathbf{u})\rangle = \frac{2}{9}\frac{\rho_p a^2 g}{\rho_f \nu} \equiv V_{TS}, \qquad [5b]$$

where V_{TS} is the settling velocity if the Stokes drag is in force or $\phi = 1$. For $V_T \gg u_0$, a Taylor series expansion can be performed for the nonlinear term on the left-hand side of [5b], which yields

$$V_T = V_{TS}\left\{1 + b\,\mathrm{Re}_1^n + bn\,\mathrm{Re}_1^n \frac{\frac{n+1}{2}\langle(v_1 - u_1)^2\rangle + \langle(v_2 - u_2)^2\rangle}{V_T^2}\right\}^{-1} \qquad [6]$$

for particles in an isotropic turbulence [see Mei (1990) for detail]. In the above, $\mathrm{Re}_1 = V_T 2a/\nu$, v_1 and u_1 are the turbulent velocity components of the particle and fluid in the direction of settling, and v_2 and u_2 are those on the direction perpendicular to gravity. The last term in the braces in

[6], which is positive, represents the effect of turbulence on the settling velocity and reduces V_T for a given V_{TS}. For V_T comparable to the turbulence or smaller, the Taylor series expansion is invalid but the qualitative feature depicted by [6] persists. Similar results were obtained for a single-component frequency oscillation of a fluid by Hwang (1985) and Ikeda & Yamasaka (1989).

To include both the effect of the trajectory bias and the effect of nonlinearity of the drag in the study of the particle motion in turbulence, a random, isotropic, Gaussian, pseudo turbulence is used and it is represented as

$$u_i(\mathbf{x}, t) = \sum_{m=1}^{N} [b_i^{(m)} \cos(\mathbf{k}^{(m)} \cdot \mathbf{x} + \omega^{(m)} t) + c_i^{(m)} \sin(\mathbf{k}^{(m)} \cdot \mathbf{x} + \omega^{(m)} t)], \qquad [7]$$

where N ($=64$ in this study) is the number of Fourier modes, and $\mathbf{k}^{(m)}$ and $\omega^{(m)}$ are the wavenumber and frequency of the mth mode. In [7], $\mathbf{k}^{(m)}$ and $\omega^{(m)}$ follow normal distributions with zero mean and variances k_0 and ω_0 (Maxey 1987), which are interpreted as the typical wavenumber and frequency of the turbulence. The random coefficients $b_i^{(m)}$ and $c_i^{(m)}$ also follow a normal distribution and are proportional to the turbulent root-mean-squared (r.m.s.) velocity u_0. The typical frequency is related to the typical wavenumber of the turbulence as $\omega_0 = \gamma k_0 u_0$, $\gamma \geqslant 0$. The energy spectrum, which is incorporated in $b_i^{(m)}$ and $c_i^{(m)}$, in this study is

$$E(k) = \frac{32 u_0^2}{\sqrt{2\pi}} \frac{k^4}{k_0^5} \exp\left(\frac{-2k^2}{k_0^2}\right), \qquad [8]$$

following Kraichnan (1970) and Maxey (1987). The details of the implementation can be found in Maxey (1987) and Mei (1990). It is noted that the above $E(k)$ lacks the $k^{-5/3}$ inertial subrange and the dynamic feature of the turbulence is not incorporated. It is only an approximation for low Reynolds number turbulence. Thus, the quantitative results in this paper are limited to low Reynolds number turbulence and the effect of the turbulence Reynolds number on the settling rate and preferential concentration is not certain at this stage. It is also noted that there is no dynamical evolution and no development of third-order correlations in the above random Fourier mode representation for turbulence. While the quantitative results should be interpreted with caution, the Monte Carlo simulation in the above form can be quite useful in demonstrating the importance of the mechanisms that affect the particle settling velocity.

By using the dimensionless variables

$$\tilde{t} = t k_0 u_0, \quad \tilde{\mathbf{Y}} = \mathbf{Y} k_0, \quad \tilde{\mathbf{v}} = \frac{\mathbf{v}}{u_0}, \qquad [9]$$

the particle dynamic equation can be made dimensionless as

$$\frac{d\tilde{\mathbf{V}}}{d\tilde{t}} = \tilde{\beta}_S[1 + b\ \mathrm{Reu}_0^n |\tilde{\mathbf{u}} - \tilde{\mathbf{V}}|^n](\tilde{\mathbf{u}} - \tilde{\mathbf{V}}) + \frac{1}{\mathrm{Fr}} \qquad [10]$$

and

$$\frac{d\tilde{\mathbf{Y}}}{d\tilde{t}} = \tilde{\mathbf{V}}. \qquad [11]$$

The dimensionelss parameters in [10],

$$\mathrm{Fr} = u_0^2 \frac{k_0}{g}, \quad \tilde{\beta}_S = \frac{9}{2} \frac{\nu \rho_f}{\rho_p a^2 k_0 u_0}, \quad \mathrm{Reu}_0 = \frac{u_0 2a}{\nu}, \quad \lambda = \frac{V_T}{u_0}, \qquad [12]$$

are the Froude number (Fr), defined in Reeks (1977) to characterize the effect of the gravitational force on the particle dispersion, the inertia parameter for Stokesian particles ($\tilde{\beta}_S$), or the ratio of the large eddy turnover time to the particle response time, the particle turbulence Reynolds number (Reu_0) and the settling rate (λ). It is noted that λ does not enter [10] and [11]; rather it is part of the solution. For a Stokesian particle settling in a still fluid $\mathrm{Reu}_0 = 0$ and the settling rate can be found by balancing the Stokesian drag with the gravity,

$$\lambda_S = \frac{1}{\mathrm{Fr}\, \tilde{\beta}_S}. \qquad [13]$$

As an indication of the nonlinearity of [10], the time-averaged particle Reynolds number is defined as

$$\mathrm{Re}_0 = \langle \mathrm{Re}_p \rangle. \tag{14}$$

Equations [10] and [11] are solved using a multistep fourth-order predictor(Adams–Bashforth)–corrector(Adams–Moulton) method. It is accurate up to $O(\Delta t^4)$. The numerical procedure can be found in Mei (1990). The ensemble averages are obtained over N_p particles and a subsequent time average is carried out over a period of $(\tilde{T}_f - \tilde{T}_i)$, in which $\tilde{T}_i$ is the instant the statistical average begins, when the particle reaches a dynamic equilibrium with the surrounding turbulence, $\tilde{T}_f$ is the total simulation time for the particle. In this study, a value $>4/\tilde{\beta}_S$ is chosen for $\tilde{T}_i$ and the typical value of $(\tilde{T}_f - \tilde{T}_i)$ is around 20 for most cases except for very large $\tilde{\beta}_S$.

The r.m.s. statistical error, ϵ_x, for a random quantity x is estimated using the following expression (Bendat & Piersol 1986):

$$\epsilon_x = \left[\frac{2\sigma_x^2 T_x}{N_p(T_f - T_i)}\right]^{1/2}; \tag{15}$$

in which σ_x^2 is the variance of x and T_x is the correlation time of x.

In the following sections, the overhead "$\sim$" will be dropped and dimensionless quantities are implicit.

3. SETTLING VELOCITY WITH LINEAR AND NONLINEAR DRAG LAWS

3.1. Analytical prediction for the settling velocity in the nonlinear drag range without trajectory bias

Mei (1990) studied analytically the dispersion of particles with a large settling rate and with zero settling in a pseudo turbulence, described by [8], by extending Reeks (1977) analysis to the nonlinear drag range. Details can be found in Mei (1990). The results that are relevant to the present study are summarized briefly below.

Because of the drag nonlinearity associated with the finite settling velocity, the particle response time constants for the particle fluctuating velocity in the directions parallel to and perpendicular to the settling, β_1^{-1} and β_2^{-1}, are different. The particle fluctuating velocity, v_α, is governed approximately by

$$\frac{dv_\alpha}{dt} = \beta_\alpha(u_\alpha - v_\alpha), \quad \alpha = 1, 2 \text{ and } 3. \tag{16}$$

In the above, it is important to interpret β_1^{-1} and β_2^{-1} as the particle response time for the fluctuating particle velocity components in the quasilinear analysis for the nonlinear problem. The constant β_α is yet to be determined as part of the solution. The validity of [16] and the introduction of β_α can be, and has been, assessed by comparing the values of the particle turbulence intensity $\langle v_\alpha^2 \rangle$ obtained in the quasilinear analysis with that obtained from the Monte Carlo simulation. For an arbitrary settling velocity, the following interpolation formulas were proposed in Mei (1990) to relate β_α, through λ, to the turbulence characteristics of the fluid and particles:

$$\mathrm{Re}_0 = \mathrm{Reu}_0\left[\lambda^2 + \frac{8}{3\pi}(3 - \langle v_1^2 \rangle - 2\langle v_2^2 \rangle\right]^{1/2} \tag{17}$$

$$\frac{\beta_1}{\beta_S} = 1 + b(1-n)\mathrm{Re}_0^n + bn\,\mathrm{Re}_0^{n-1}\,\mathrm{Reu}_0\left[(2\lambda)^2 + \frac{8}{3\pi}(3 - \langle v_1^2 \rangle - 2\langle v_2^2 \rangle)\right]^{1/2} \tag{18a}$$

and

$$\frac{\beta_2}{\beta_S} = 1 + b\,\mathrm{Re}_0^n. \tag{18b}$$

The settling velocity is finally calculated from

$$\lambda = \frac{1}{\mathrm{Fr}\,\beta_2}. \tag{19}$$

The analytical expression for the particle intensity $\langle v_\alpha^2 \rangle$ in a turbulence described by [8] for a given set of λ and β_α is obtained from the extended Reeks' (1977) solution (Mei 1990). The solutions for $\langle v_\alpha^2 \rangle$, β_1, β_2, Re_0 and λ can be obtained through an iterative procedure. Equations [17]–[19] include the effect of turbulence associated with the nonlinear drag but neglect the trajectory bias effect completely. The above treatment for evaluating β_α is not exact; but it has been verified by comparing the results from the Monte Carlo simulation over a range of parameters.

For a given particle, there are several settling velocities that can be defined, each depending on the turbulence environment, characterized by Reu_0, and the particle Reynolds number Re_0 which determines whether the drag law is linear or nonlinear:

(i) For particle settling in a still fluid ($\mathrm{Reu}_0 = 0$), one obtains $\lambda_1 = \lambda_S$ defined by [13] if the Stokes drag is in force ($\mathrm{Re}_0 \ll 1$). This settling rate, λ_S, can always be defined for convenience even if $\mathrm{Re}_0 \sim 1$ or larger and the particle settles in a turbulent flow.

(ii) For $\mathrm{Reu}_0 \ll 1$ and $\mathrm{Re}_0 \ll 1$, the Stokes drag is still in force but turbulence will enhance the settling velocity, as found by Maxey (1987). This settling velocity is denoted by λ_2.

(iii) In a still fluid with $\mathrm{Reu}_0 = 0$ but $\mathrm{Re}_0 \sim O(1)$ or larger, the settling velocity is denoted by λ_3; and it can be obtained simply by solving the nonlinear algebraic equation [6]. This settling velocity can be achieved if a particle settles in a relatively weak turbulence with $\lambda = V_T/u_0 = \mathrm{Re}_0/\mathrm{Reu}_0 \gg 1$; hence the settling is not affected by turbulence. For the same β_S, $\lambda_3 > \lambda_1$, as seen from [6], because of the nonlinearity. Therefore, a comparison of λ_S between the cases of linear and nonlinear drag under the same β_S is not meaningful. Rather, an effective particle inertia parameter should be used in the linear case.

(iv) For $\mathrm{Reu}_0 \sim O(1)$ and $\mathrm{Re}_0 \sim O(1)$, the nonlinear analysis of the particle dispersion described by Mei (1990) gives $\lambda = \lambda_4$ and it is computed from [19]. The difference between λ_3 and λ_4 is due to the effect of turbulence in the nonlinear drag range. In the limit of small Reu_0, λ_4 recovers λ_3.

(v) For $\mathrm{Reu}_0 \sim O(1)$ and $\mathrm{Re}_0 \sim O(1)$, the Monte Carlo simulation outlined in section 2 includes both the effects of the nonlinear drag and the trajectory bias. The resulting settling velocity is denoted as λ_5. This settling velocity is physically the most realistic one, but its accuracy is negated by the statistical error of the Monte Carlo simulation. Therefore, a large number of particles are required.

3.2. Results and discussions

For $\lambda = \lambda_2$, Maxey (1987) has shown that the effect of the particle trajectory bias is the strongest when $\gamma = \omega_0/(k_0 u_0) = 0$, which corresponds to a frozen turbulence. As γ increases, the eddy self-decay becomes important and particles are less likely to accumulate in regions of high strain rate and low vorticity. At $\lambda_S = 1$, the increase in the settling velocity, $\lambda_2 - \lambda_S$, was 0.068 to 0.082 for $\beta_S = 1$ and 0.08 to 0.1 for $\beta_S = 4$ based on the simulation of $N_p = 1000$ particles (Maxey 1987). The present Monte Carlo simulation using a linear drag gives $\lambda - \lambda_S = 0.079$ and 0.083 for $\beta_S = 1$ and $\beta_S = 4$ in runs with $N_p = 5000$ particles. Good agreement is observed between the present results and that of Maxey. $N_p = 5000$ is chosen so that the statistical error in the present simulation is less than that in the simulation by Maxey (1987), and a meaningful comparison can be made. Thus, the present Monte Carlo simulation is validated with regard to the effect of turbulence on the settling velocity in the Stokes drag range.

In order to compare meaningfully the settling velocity based on a linear drag with that based on a nonlinear drag and to assess the effect of the trajectory bias, nearly the same effective particle inertia parameter should be used. For non-Stokesian particles, the effective particle inertia parameter is $\beta_2 \sim \beta_S(1 + b\,\mathrm{Re}_0^n)$. Since Re_0 is affected by the turbulence structure, β_2 is not a constant as γ varies. For meaningful comparisons of different λs, $\mathrm{Reu}_0 = 4$ and $\mathrm{Fr} = 1$ are chosen first. Through trial-and-error, $\beta_S = 0.6667$ gives $\lambda_4 \approx 1$ (actual value: 0.9975) and $\beta_2 \approx 1$ (actual value: 1.0025) at $\gamma = 1$ by using the nonlinear analysis (Mei 1990). An order unity λ is desirable because the trajectory bias effect is the strongest near $\lambda \sim 1$ for $\beta_S \sim 1$. This group of parameters give $\lambda_3 = 1.0666$ when the effect of turbulence is neglected completely. In computing λ_2 for Stokesian particles, $\lambda_S = 1$ requires $\beta_S = 1$, instead of $\beta_S = 0.6667$.

Figure 1 compares λ_2, λ_3, λ_4 and λ_5 as a function of γ for $\mathrm{Reu}_0 = 4$ and $\mathrm{Fr} = 1$. For λ_3, λ_4 and λ_5, $\beta_S = 0.6667$ is used; while for λ_2, $\beta_S = 1$. The error bars for λ_4 and λ_5 are obtained using [15] for $x = v_1$; the product of the variance and the correlation time is simply the particle diffusivity in the gravitational direction. For $\beta_S \approx 1$, the simulations are carried out with $(T_f - T_i) = 20$. The number of particles used in the simulation to obtain λ_5 ranges from $N_p = 5000$ to 31,000 (for $\gamma = 1$). For Monte Carlo simulation, the results are obtained using Stokesian drag, $N_p = 12{,}000$, 15,000 and 10,000 for $\gamma = 0.5$, 0.6 and 0.7; the rest are obtained using $N_p = 4000$ particles. They are presented to assist the interpretation of the results based on the nonlinear simulation.

In figure 1, first note the comparison between the settling velocity in the still fluid ($\mathrm{Reu}_0 = 0$), λ_3, and the actual settling velocity from the nonlinear simulation, λ_5, for the same particle. The effect of the turbulence on the settling velocity in the nonlinear drag range is clearly represented because $\Delta\lambda = \lambda_5 - \lambda_3$ contains two opposing effects of the turbulence: nonlinearity of the drag and the trajectory bias. It can be seen that the overall effect of the turbulence reduces the settling velocity, except near $\gamma = 0$. This is in contrast to the linear drag case in which the turbulence enhances the settling velocity. It appears that the effect of the trajectory bias is stronger than the effect of nonlinearity for $\gamma \leqslant 0.1$, but it is offset to a large extent by the nonlinearity of the drag for $\gamma > 0.1$. As γ increases, the Eulerian turbulence time scale decreases—which makes it more difficult for the particle to follow fluid turbulence. As a result, $\langle v_\alpha^2 \rangle$ decreases, β_2 increases and λ decreases, as indicated in the nonlinear analysis by Mei (1990) or [17]–[19]. Furthermore, as Maxey (1987) found, an increasing γ results in a decrease in the trajectory bias effect on the settling velocity; the trajectory bias effect practically vanishes beyond $\gamma = 1$. This trend agrees with the results of the present nonlinear simulation. Thus, for $\gamma \sim O(1)$, the effect of the nonlinear drag is more important than the trajectory bias effect. Since the present result is for $\mathrm{Re}_0 = 5.62$, which is a relatively low Reynolds number compared with the larger values in the experiments cited in Hwang (1985), the effect of the drag nonlinearity should be more dominant.

To sort out the effect of the trajectory bias in the nonlinear drag case, we examine closely the nonlinear simulation result at $\gamma = 0$, which gives $(\lambda_5, \beta_2) = (1.091, 0.9763)$, and at $\gamma = 1$, which gives $(\lambda_5, \beta_2) = (0.9764, 0.9888)$. As has been discussed in the introduction, there are two mechanisms for the change in λ: (i) the change in β_2 due to the nonlinearity in the drag; and (ii) the trajectory bias.

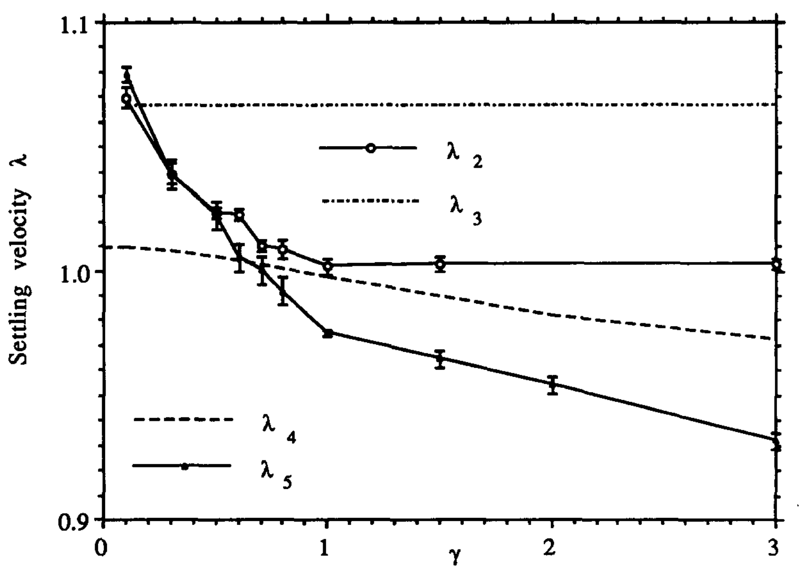

Figure 1. Effect of the turbulence structure (γ) on the particle settling velocity (λ) using the Stokes drag with ($\beta_S = 1$, $\lambda_S = 1$) and the nonlinear drag law with $\mathrm{Reu}_0 = 4$, $\beta_S = 0.6667$ and $\mathrm{Fr} = 1$: λ_2—Monte Carlo simulation using the linear drag law; λ_3—still fluid with the nonlinear drag law; λ_4—nonlinear analysis without the trajectory bias effect; λ_5—Monte Carlo simulation using the nonlinear drag law and including both the effects of the trajectory bias and the nonlinearity of the drag.

Because β_2 changes as well in the two simulations ($\gamma = 0$ and $\gamma = 1$), the change in λ could result from both mechanisms. However, it is observed that the relative change in β_2 is $\Delta\beta_2/\beta_2 = 0.013$ from $\gamma = 0$ to 1, while the relative change in the settling rate is $\Delta\lambda/\lambda = -0.117$. If one uses $\lambda = 1/(\mathrm{Fr}\beta_2)$ to estimate the relative change in λ caused by the decrease in β_2, $\Delta\lambda/\lambda = -\Delta\beta_2/\beta_2 = -0.013$ would be obtained. The fact that $\Delta\lambda/\lambda = -0.117 \gg 0.013$ shows that the decrease in λ as the turbulence structure changes from $\gamma = 0$ to 1 is not caused by the increase in β_2. Rather, the effect of the trajectory bias is responsible for the change in $\Delta\lambda$ in the nonlinear drag case. The similarity in the values of λ_2 and λ_5 at small γ indicates that the trajectory bias is in force in both the linear and nonlinear drag cases.

It is also seen in figure 1 that the settling velocity based on the nonlinear anlaysis, λ_4, is smaller than that of the simulation λ_5 for small values of λ. This arises because the nonlinear analysis does not take the trajectory bias into account. Thus, the difference $(\lambda_4 - \lambda_5)$ may be interpreted as due to the trajectory bias effect for small γ. For $\gamma > 0.7$, the nonlinear analysis overpredicts, by a few percent, the settling velocity λ_5. Noticing that $(\lambda_4 - \lambda_5)$ is smaller than $(\lambda_3 - \lambda_4)$, which is mainly due to the nonlinearity of the drag associated with the fluctuation of the velocity, it is appropriate to say that for $\gamma = 1$ the effect of the trajectory bias is small and that the nonlinear analysis can be used to assess the effect of turbulence on the settling velocity satisfactorily.

It should be pointed out that the settling velocity λ_2 shown in figure 1 is obtained from the Monte Carlo simulation using a linear drag with assumed $\beta_S = \beta_2 \approx 1$ and $\lambda_S = 1$ as inputs. However, the effective particle inertia parameter β_2 in the nonlinear drag range is not known *a priori* and it should be obtained as part of the solution. A traditional method for determining β_2 is to solve for λ_3 first by assuming $u_0 = 0$. This gives $\lambda_3 = 1.0666$ and $\beta_2 = 1/\lambda_3 = 0.9376$, which are not accurate because the turbulence effects are completely neglected. Had this set of data for $(\beta_2, \lambda) = (0.9376, 1.0666)$ been used in the linear simulation, the λ_2-curve in figure 1 would be shifted upward to surpass the λ_3-curve by about 6%. From this view point, the result from the nonlinear analysis, λ_4, is better than λ_2 in comparison with λ_5 in the nonlinear drag range for $\gamma \sim O(1)$.

Figure 2 compares the settling velocity obtained from the nonlinear analysis given in Mei (1990) with that from the Monte Carlo simulation for $\mathrm{Reu}_0 = 4$ and $\mathrm{Fr} = 1$ for $\gamma = 1$ over a range of β_S using $N_p = 4000$. It can be seen that for $\gamma = 1$, the nonlinear analysis without the trajectory bias effect gives a very reliable result. The largest absolute difference occurs, for $0.234 < \beta_S < 7.475$, at $\lambda_5 = 2.364$ with $\lambda_4 - \lambda_5 = 0.036$. This comparison indicates, again, that for a nonfrozen turbulence, the effect of the trajectory bias can be overwhelmed by the nonlinearity of the drag law. Shown in figure 2(b) is the corresponding particle Reynolds number, Re_0, which indicates the degree of nonlinearity in the drag law, for $0.234 < \beta_S < 7.475$ and for $\mathrm{Reu}_0 = 4$ and $\mathrm{Fr} = 1$ at $\gamma = 1$. Since

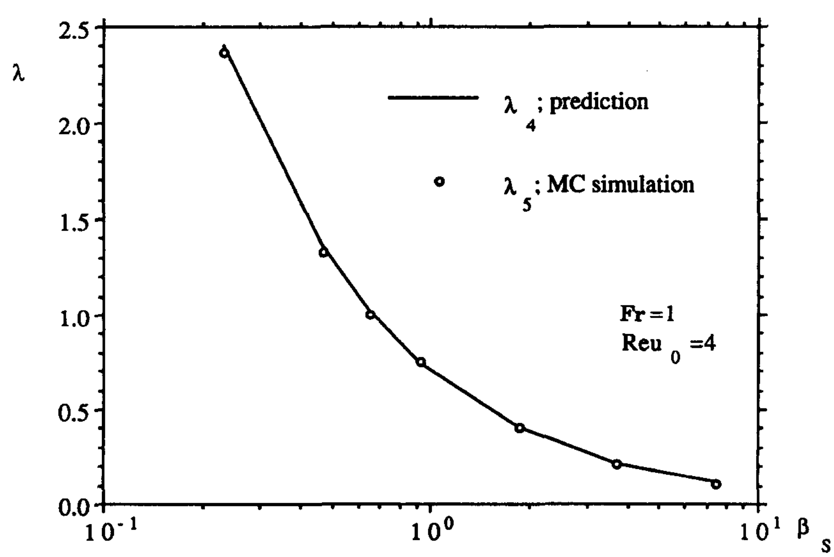

Figure 2(a). Comparison of the settling velocity between the nonlinear prediction λ_4 and the Monte Carlo simulation λ_5 for $\mathrm{Reu}_0 = 4$, $\mathrm{Fr} = 1$ and $\gamma = 1$.

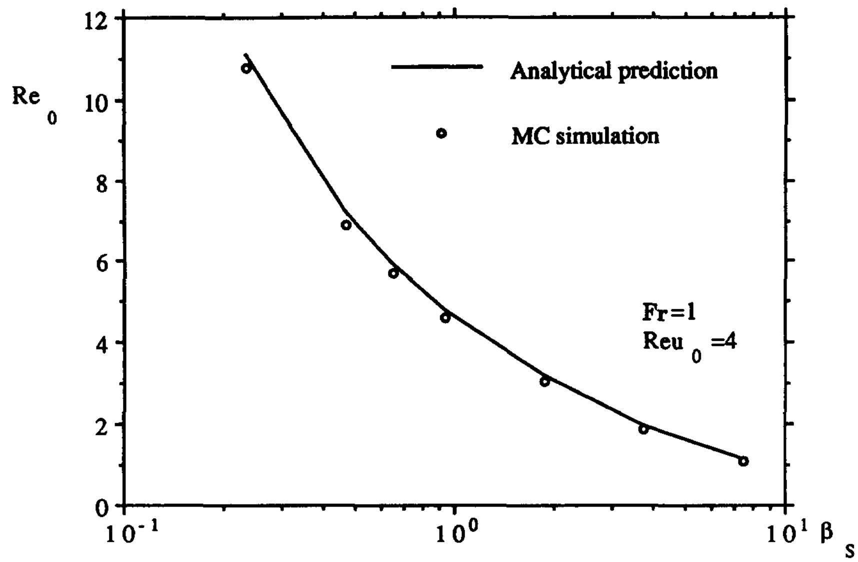

Figure 2(b). Variation of the particle Reynolds number Re_0 as a function of β_S for $Reu_0 = 4$, $Fr = 1$ and $\gamma = 1$.

$Re_0 > 1$ for $\beta_S < 7.475$ in figure 2(b), it is clear that the nonlinearity of the drag must be incorporated in the particle dispersion analysis. Otherwise, an overprediction in λ would occur. In the cases considered by Hwang (1985), $Re_0 \gg 1$; hence the reduction in V_T results from the drag nonlinearity, whose effect is much more important that the trajectory bias.

4. SIMULATION RESULTS FOR THE SECOND INVARIANT OF THE TURBULENCE DEFORMATION TENSOR ON THE PARTICLE TRAJECTORY

Maxey (1987) showed that the trajectory bias is caused by the accumulation of particles in regions of high strain rate or low vorticity. If particles are treated as a continuum with velocity $\mathbf{v}(t, \mathbf{x})$, the equation governing the distribution of the particle number density $n(t, x, y, z)$ can be expressed as

$$\frac{\mathrm{d}n}{\mathrm{d}t} \equiv \frac{\partial n}{\partial t} + \mathbf{v} \cdot \nabla n = -n\nabla \cdot \mathbf{v}. \tag{20}$$

By integrating this equation with respect to time, it is seen that particles accumulate in regions of large negative $\nabla \cdot \mathbf{v}$. For particles with weak inertia, Maxey (1987) has shown that

$$\nabla \cdot \mathbf{v} \approx \beta_S^{-1} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}, \quad \text{for } \beta_S \gg 1, \tag{21}$$

to the leading order in particle inertia β_S^{-1}. Since

$$\mathrm{II_d} = -\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} \times \frac{\partial u_j}{\partial x_i}\right) = -\frac{1}{2}\left(S^2 - \frac{\Omega_j \Omega_j}{4}\right) \tag{22}$$

is the second invariant of the deformation tensor of the fluid turbulence, where S is the magnitude of the strain rate tensor $s_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$ and Ω_j is the jth component of the vorticity, it follows that

$$\frac{\mathrm{d}n}{\mathrm{d}t} \approx \frac{-2\mathrm{II_d}}{\beta_S}, \quad \text{for } \beta_S \gg 1. \tag{23}$$

Hence, the effect of the trajectory bias found by Maxey (1987) is directly related to $\mathrm{II_d}$ evaluated on the particle trajectory. It can be used as a measure of the trajectory bias or preferential concentration because particles tend to collect in regions of large negative $\mathrm{II_d}$. Squires & Eaton (1990) demonstrated, in the absence of settling, that there are similarities between the instantaneous contours of particle number density and that of $\mathrm{II_d}$ in their DNS. As a result, the particle concentration could be quite nonuniform in space and the settling velocity would increase.

To examine further the effect of the turbulence structure on the particle settling velocity and the particle concentration (or number density) distribution, the ensemble average of the second invariant $\langle \mathrm{II_d} \rangle$, which has been scaled by $u_0^2 k_0^2$, seen by the particle is computed from the Monte Carlo simulation using the linear and nonlinear drag laws for various parameters. Figure 3(a) shows $\langle \mathrm{II_d}(t) \rangle$ for $\beta_S = 1$, $\gamma = 0$, $\lambda_S = 0$, 1 and 2 from the simulations with linear drag using $N_p = 15{,}000$ particles in each case. It can be seen that the largest magnitude of $\langle \mathrm{II_d}(t) \rangle$ occurs when $\lambda_S = 0$. At $\lambda_S = 2$, $\langle \mathrm{II_d}(t) \rangle$ oscillates in time with a much smaller average value. This implies that when the settling velocity is small, particles will be able to find regions of large strain rate or low vorticity. As a result, there will be a correlation between the particle concentration and $\mathrm{II_d}(t, x, y, z)$, as shown in Squires & Eaton (1990). As λ_S increases, the particle trajectory becomes straighter, so the ability of the particle to find and reside in regions of large strain rate diminishes. Figure 3(b) shows the dependence of time-averaged $\langle \mathrm{II_d} \rangle$ on the particle time constant β_S for $\gamma = 1$ and $\lambda_S = 0$ for β_S between 1 and 50. The statistical error of $\langle \mathrm{II_d}(t) \rangle$ is evaluated from [15] with the variance of $\mathrm{II_d}$ near 0.9. The correlation time for $\mathrm{II_d}$ is around 0.7 for $\lambda_S = 0$; it decreases with increasing λ_S. The error for $\beta_S = 50$ is relatively large because $(T_f - T_i) = 2$, due to the much smaller Δt used for stability reasons. It is noted from the data that the maximum value of $\langle \mathrm{II_d} \rangle / \beta_S$ occurs near $\beta_S = 1$ and $\langle \mathrm{II_d} \rangle / \beta_S$ decreases as β_S increases. This implies a loss of correlation between $n(t, x, y, z)$ and $\langle \mathrm{II_d}(t, x, y, z) \rangle$ as seen from [23]. Comparing figures 3(a) and 3(b), it is seen that $\langle \mathrm{II_d}(\gamma = 1, \beta_S = 1, \lambda = 1) \rangle$ is about 4 times smaller than $\langle \mathrm{II_d}(\gamma = 0, \beta_S = 1, \lambda = 1) \rangle$. This indicates the importance of the Eulerian time scale, which characterizes the self-decay of turbulent eddies on $\langle \mathrm{II_d} \rangle$. Figure 3(c) shows the dependence of $\langle \mathrm{II_d} \rangle$ on λ_S for $\beta_S = 1$ and $\gamma = 1$ [cf. figure 3(a) for $\gamma = 0$]. Although the relative statistical error is larger for $\lambda_S \geqslant 3.5$, it can be seen that $\langle \mathrm{II_d} \rangle$ decreases exponentially as $-\langle \mathrm{II_d} \rangle \sim 0.145 \exp(-1.233\lambda_S)$ for $\lambda_S \geqslant 1$. Hence, settling reduces the particle trajectory bias and effectively destroys the correlation between the concentration and turbulence structure.

The difference between $\langle \mathrm{II_d}(t) \rangle$ obtained from the linear and nonlinear simulations, based on 15,000 particles, with the same effective particle inertial parameter $\beta_2 = \beta_S$ at the same γ and λ is not significant. At $\beta_2 = 1$, $\gamma = 1$ and $\lambda = 1$, the nonlinear simulation gives a $\langle \mathrm{II_d} \rangle$ that is 10% larger than that of the linear simulation, which is within or around the statistical error of the simulation for $\langle \mathrm{II_d} \rangle$. The value of $\langle \mathrm{II_d}(t) \rangle$ calculated from the nonlinear simulation over 15,000 particles has larger statistical oscillations in comparison with the linear case at otherwise the same conditions. This may be attributed to the nonlinearity of the drag law, which slightly increases the high frequency energy of the particle turbulence. Thus, the previous discussion regarding the effect of λ and β_S on $\langle \mathrm{II_d} \rangle$ in the linear drag range remains valid in the nonlinear drag range.

5. CONCLUSIONS

(i) The particle settling velocity in turbulence in the nonlinear drag range is affected by both the trajectory bias and the drag nonlinearity. These two mechanisms have opposite effects. The effect of the trajectory bias on the settling velocity is important for an almost frozen turbulence and $\lambda \sim 1$. The existence of the trajectory bias is also demonstrated in the nonlinear drag range. On the other hand, for a nonfrozen, low Reynolds number turbulence near $\gamma = 1$, the effect of the trajectory bias on the settling may be overwhelmed by the effect of the drag nonlinearity associated with the turbulence if the particle Reynolds number is large enough.

(ii) The average value of the second invariant of the turbulence deformation tensor, $\langle \mathrm{II_d} \rangle$, evaluated on the particle trajectory, can be used to describe the trajectory bias. It attains its maximum value at $\lambda = 0$ for a given β_2; it decays exponentially for $\lambda \geqslant 1$ and almost vanished for $\lambda \sim 3$. It also vanishes for very large or small values of β_2; the maximum of

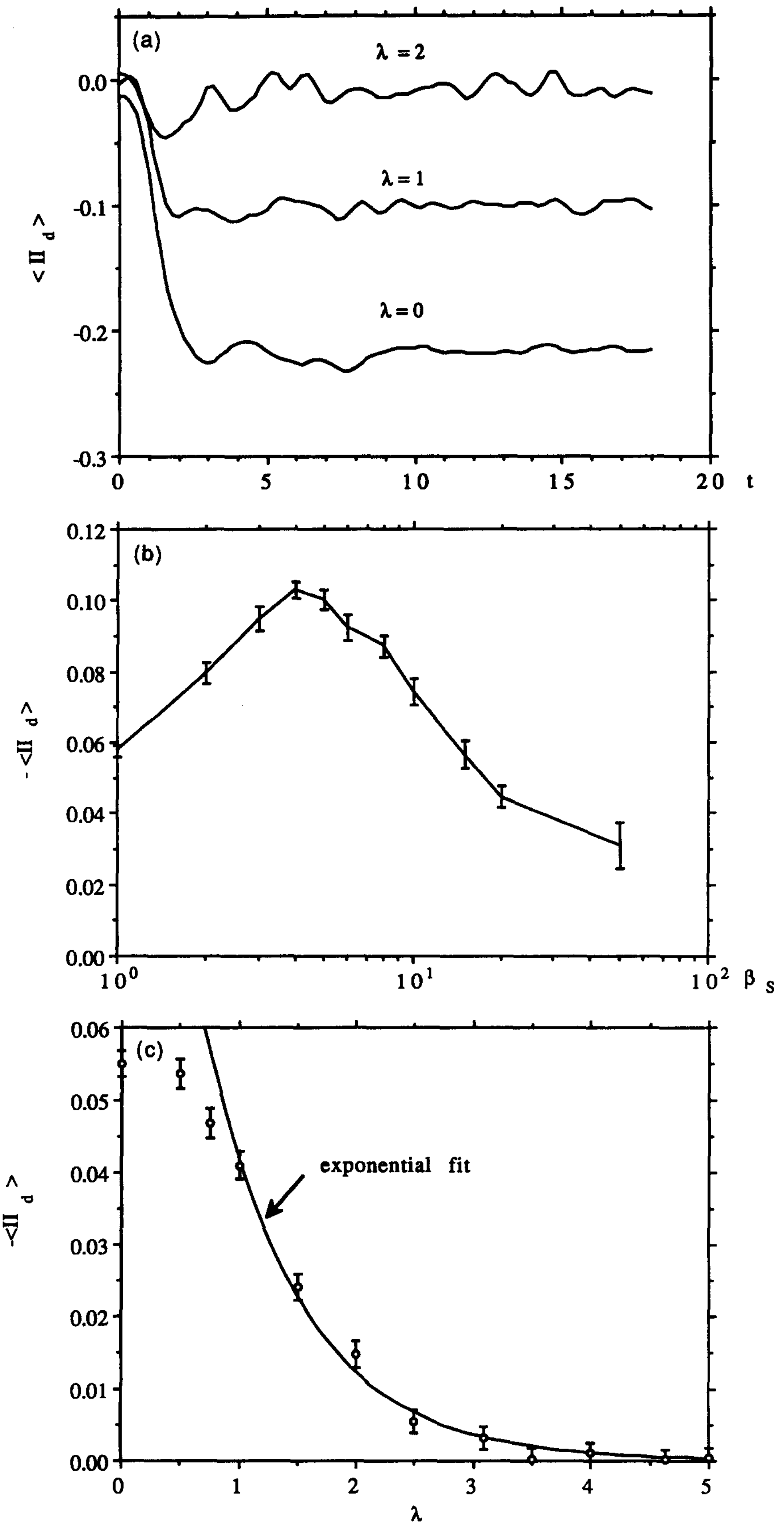

Figure 3. Ensemble average of $\langle II_d \rangle$ ($N_p \geq 15{,}000$) using the Stokes drag: (a) dependence of $\langle II_d \rangle$ on λ_S at $\beta_S = 1$ in a frozen turbulence ($\gamma = 0$); (b) dependence of $\langle II_d \rangle$ on β_S at $\gamma = 1$ and $\lambda_S = 0$; (c) dependence of $\langle II_d \rangle$ on λ_S at $\beta_S = 1$ in a nonfrozen turbulence ($\gamma = 1$).

$\langle II_d \rangle$ occurs at order unity β_2 for a given λ and γ. For the same values of λ and β_2, there is no essential difference in $\langle II_d \rangle$ between using linear drag or nonlinear drag.

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